CS482

Towards Monte Carlo Physically-Based Sound Synthesis

Final Presentation

Team 1

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Overview

1. Motivation
2. Method
3. Experiment
4. Conclusion

Motivation

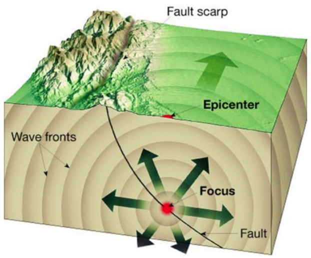
Motivation - Wave Equation is Everywhere

Wave equation : second-order linear PDE

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0 \qquad \left(\Delta - \frac{1}{c^2}\partial_t^2\right) p(x) = 0$$

Seismic Wave

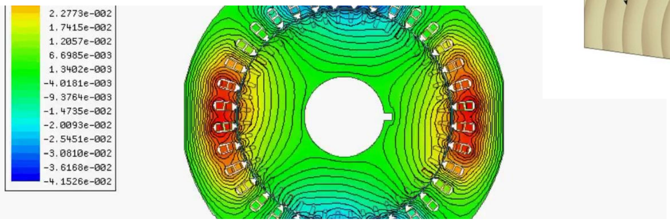
$$(\lambda + 2\mu)\nabla\nabla \cdot u - \mu\nabla \times (\nabla \times u) + \rho\omega^2 u + f = 0$$



EM Wave

$$\left(v_{\mathrm{ph}}^2 \nabla^2 - \frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \mathbf{0}$$

$$\left(v_{\mathrm{ph}}^2 \nabla^2 - \frac{\partial^2}{\partial t^2}\right)\mathbf{B} = \mathbf{0}$$

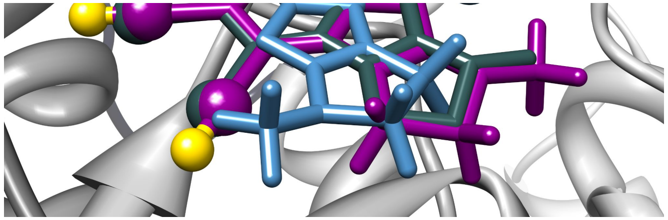


Gravitational Wave

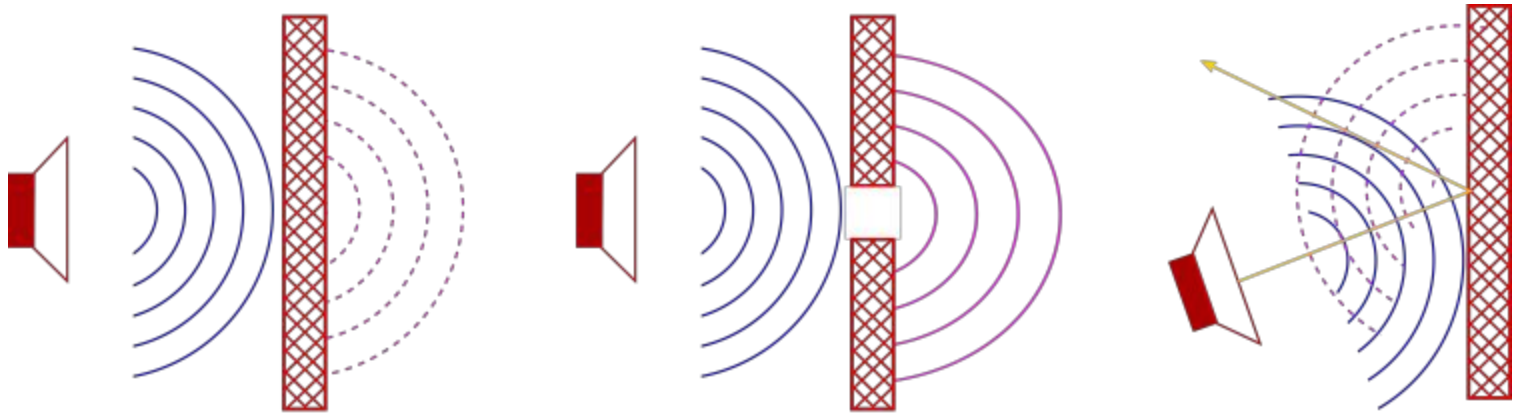
$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) h_{\mu\nu} = 0$$

Schrodinger's Equation

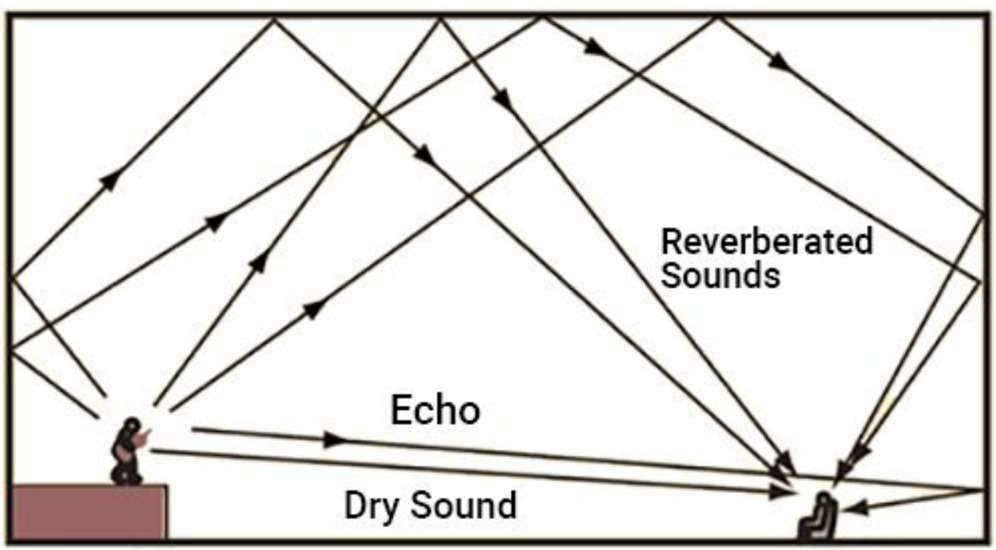
$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H} |\Psi\rangle$$



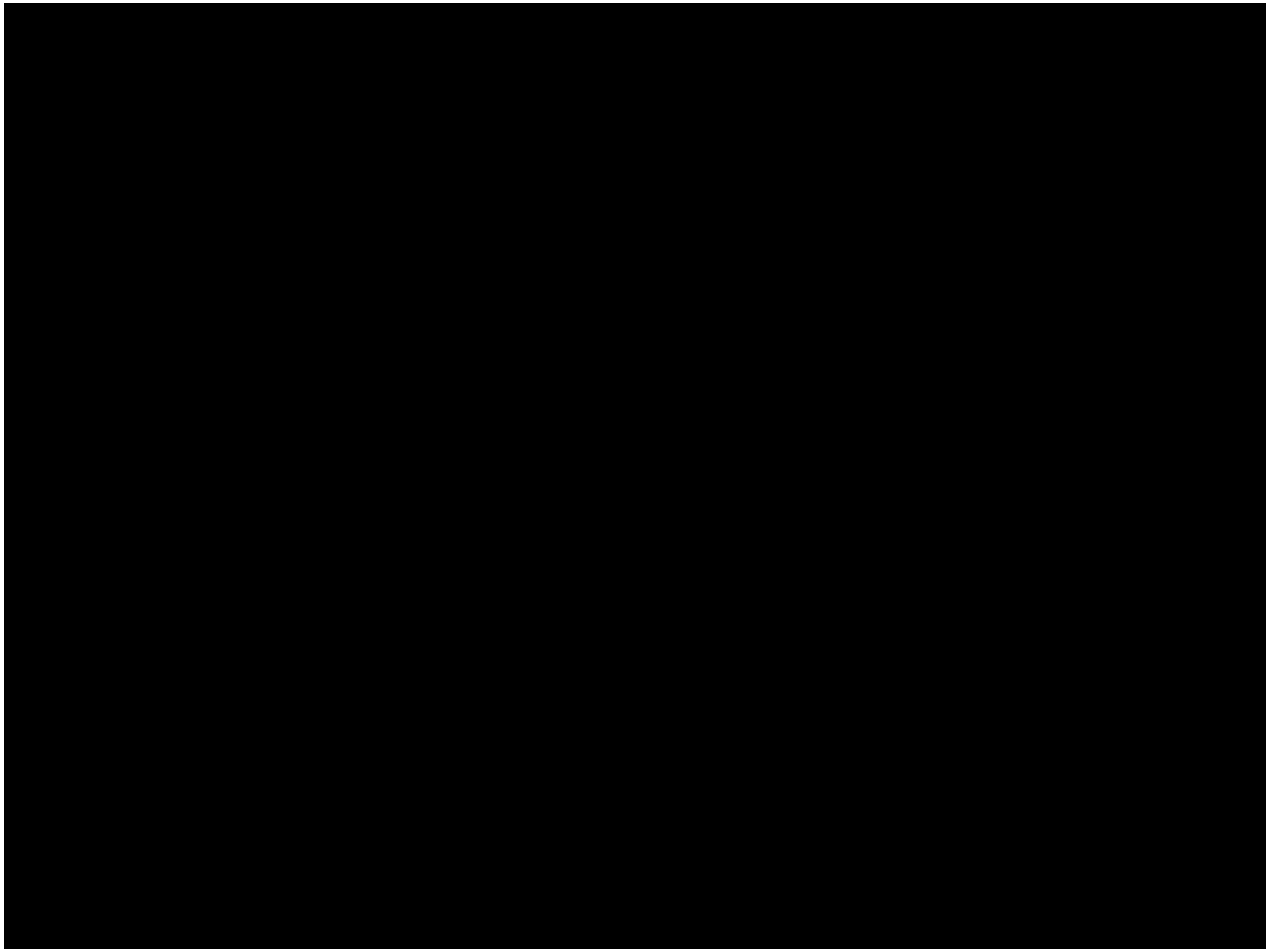
Motivation - Why Physically-based Sound (1)



Refraction Diffraction Reflection



Motivation - Why Physically-based Sound (2)



Motivation - Why Physically-based Sound (3)

Method

Background - Rigid Body Sound (Modal Sound)

Linear Elasticity (How object vibrate)

$$\mathbf{M}\ddot{u} + \mathbf{C}\dot{u} + \mathbf{K}u = \mathbf{F} \quad \text{... (1)}$$

$$\mathbf{KU} = \mathbf{\Lambda MU}, \text{ where } \mathbf{\Lambda} = \text{diag}(\omega_i^2) \quad \text{... (2)}$$

$$(1), (2) \rightarrow \ddot{q}_i + (\alpha + \beta\omega_i^2)\dot{q}_i + \omega_i^2 q = Q_i(t) \quad \because u = \mathbf{U}q$$

Acoustic Wave Equation (How sound propagate)

$$\left(\Delta - \frac{1}{c^2}\partial_t^2\right) p(x) = 0$$

Sound Render (How person hear)

$$|p(\mathbf{x})|q(t)$$

Background - Rigid Body Sound (Modal Sound)

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Volumetric FEM

$$(1), (2) \rightarrow \ddot{q}_i + (\alpha + \beta\omega_i^2)\dot{q}_i + \omega_i^2 q = Q_i(t) \quad \because u = \mathrm{U}q$$

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BEM

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Monte Carlo

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Linear Elasticity (How object vibrate)

$$M\ddot{u} + C\dot{u} + Ku = F \quad \dots (1)$$

$$KU = \Lambda MU, \text{ whe}\dots \quad \dots (2)$$

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~~**Volumetric FEM**~~ **Monte Carlo**

Acoustic Wave Equation (How sound propagate)

$$\left(\Delta - \frac{1}{c^2}\partial_t^2\right)p(x) = 0$$

~~**BEM**~~ **Monte Carlo**

Sound Render (How person hear)

$$|p(\mathbf{x})|q(t)$$

Solving Acoustic Wave

Acoustic Wave Equation (Time Domain)

$$\left(\Delta - \frac{1}{c^2}\partial_t^2\right) u(\mathbf{x}) = 0$$

Still **IMPOSSIBLE** to solve with WoS scheme

Solving Acoustic Wave

Acoustic Wave Equation (Frequency Domain)

$$\left(\Delta + \frac{\omega^2}{c^2}\right) u(\mathbf{x}) = 0$$

theoretically **POSSIBLE** to solve with WoS scheme. Not sure the implementation

Solving Acoustic Wave

Acoustic Wave Equation (Frequency Domain)

$$\left(\Delta + k^2\right) u(\mathbf{x}) = 0$$

Stochastic Representation of Wave Equation

$$u(\mathbf{x}) = \mathbb{E}\left[e^{-\frac{1}{2}k^2\tau} f(\mathbf{W}_\tau)\right]$$

Solving Acoustic Wave

Acoustic Wave Equation (Frequency Domain)

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$$u(\mathbf{x}) = \mathbb{E}\left[e^{-\frac{1}{2}k^2\tau} f(\mathbf{W}_\tau)\right]$$

very HARD to estimate termination time [Killing WoS, Weighted WoS]

Solving Acoustic Wave

Acoustic Wave Equation (Frequency Domain)

$$\left(\Delta + k^2\right) u(\mathbf{x}) = 0$$

Duffin's Correspondence

$$U(\mathbf{x}, z) = \cosh(kz)u(\mathbf{x})$$

Solving Acoustic Wave

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Wave Equation after Duffin's Transform

$$\Delta U(\mathbf{u}, z) = 0$$

Solving Acoustic Wave

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$$U(\mathbf{x}, z) = \cosh(kz)u(\mathbf{x})$$

Wave Equation after Duffin's Transform

Laplace Equation!

$$\Delta U(\mathbf{u}, z) = 0$$

Solving Acoustic Wave - Neumann BC

Acoustic Wave Equation with Neumann BC

$$(\Delta + k^2)u(\mathbf{x}) = 0, x \in \Omega$$

$$\partial_{\mathbf{n}} u(\mathbf{x}) = f(\mathbf{x}), x \in \partial\Omega$$

Duffin's Correspondence with Neumann BC

$$\Delta U(\bar{\mathbf{x}}) = 0, \bar{x} = (\mathbf{x}, z) \in \Omega \times \mathbb{R}$$

$$\partial_{\mathbf{n}} U(\bar{\mathbf{x}}) = \cosh(kz) f(\mathbf{x}), \bar{x} \in \partial\Omega \times \mathbb{R}$$

Solving Acoustic Wave - Neumann BC

Acoustic Wave Equation with Neumann BC

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Solving Acoustic Wave - Overview

Duffin Walk-on-Sphere (Dirichlet BC)

```
X = (x,0);  // Duffin's Initialization
while (dist2boundary(X) > EPS) {
    X += dist2boundary(X) * sample_s_n_plus_one();
}
return f(X[:-1]) * cosh(k*X[-1]);
```

General Duffin Walk-on-Sphere Algorithm

1. initialize $\bar{\mathbf{x}} = (\mathbf{x}, 0), \mathbf{x} \in \Omega \subset \mathbb{R}^N$
2. solve Laplace equation $\Delta U(\bar{\mathbf{x}}) = 0$
3. get $u(\mathbf{x}) = U(\bar{\mathbf{x}})$

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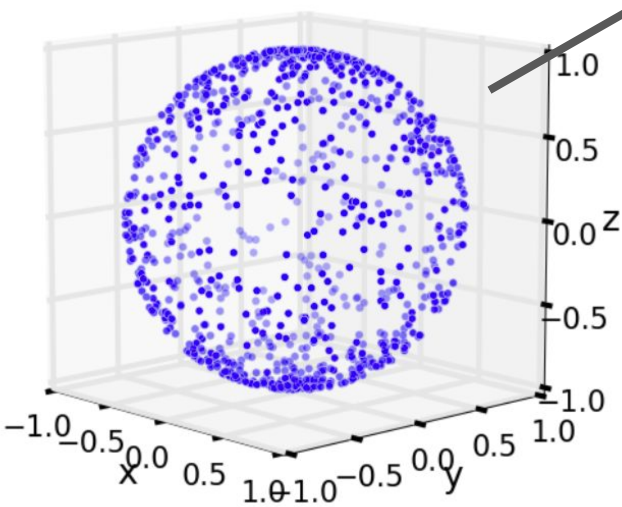

Sample Sphere of dimension N + 1

$$\mathbf{v} \sim \mathcal{N}(0, \mathbf{I}), \mathbf{v} \in \mathbf{R}^{N+1}$$

$$\mathbf{v}_{\text{normalized}} = \mathbf{v}/|\mathbf{v}|$$

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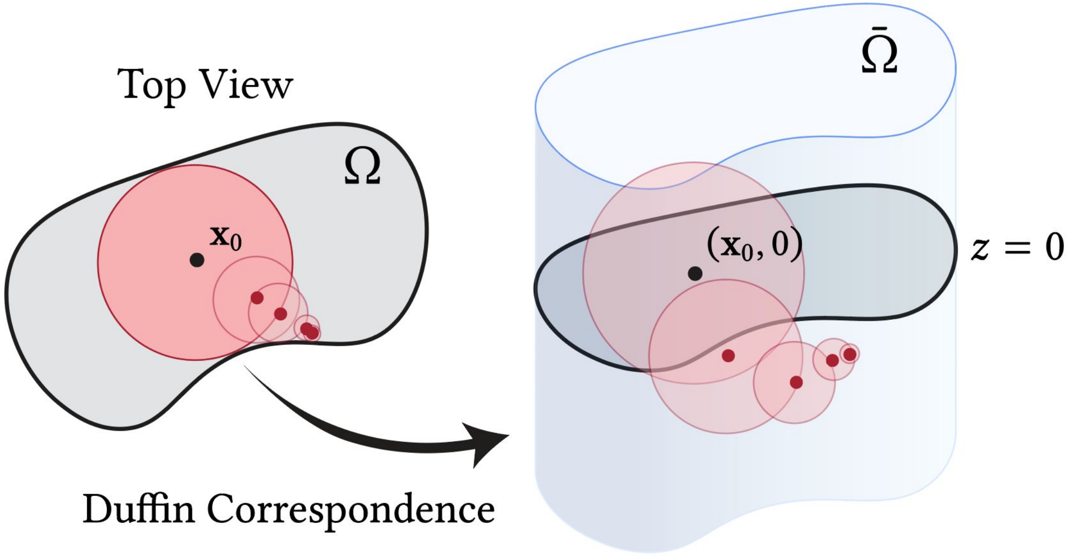

Review - Rigid Body Sound (Modal Sound)

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~~**BEM**~~

Monte Carlo

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Solving Elastic Wave

Elastic Wave Equation (Time Domain)

$$(\lambda + 2\mu)\nabla\nabla \cdot u(\mathbf{x}, t) - \mu\nabla \times (\nabla \times u(\mathbf{x}, t)) - \rho\partial_t^2 u(\mathbf{x}, t) + f(\mathbf{x}, t) = 0$$

obviously **IMPOSSIBLE** to solve with WoS scheme

Solving Elastic Wave

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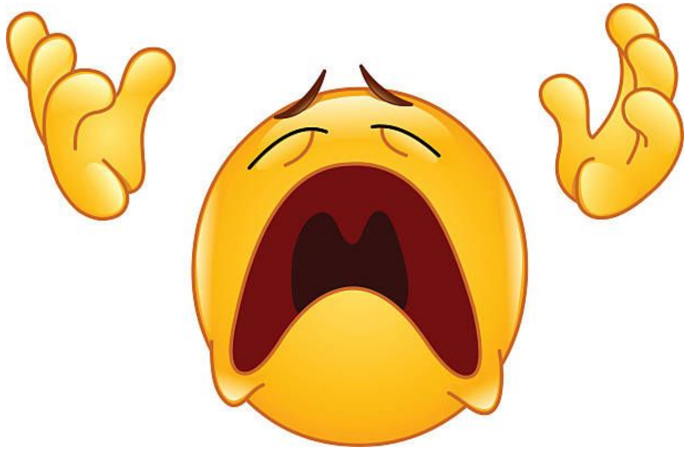

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Elastic Wave Equation (Frequency Domain)

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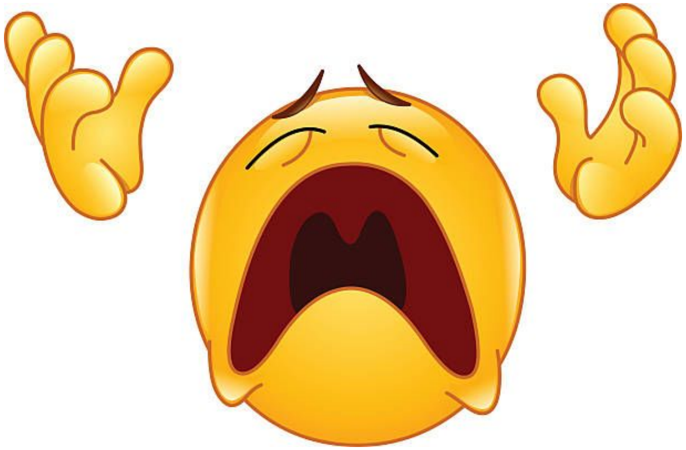


Solving Elastic Wave

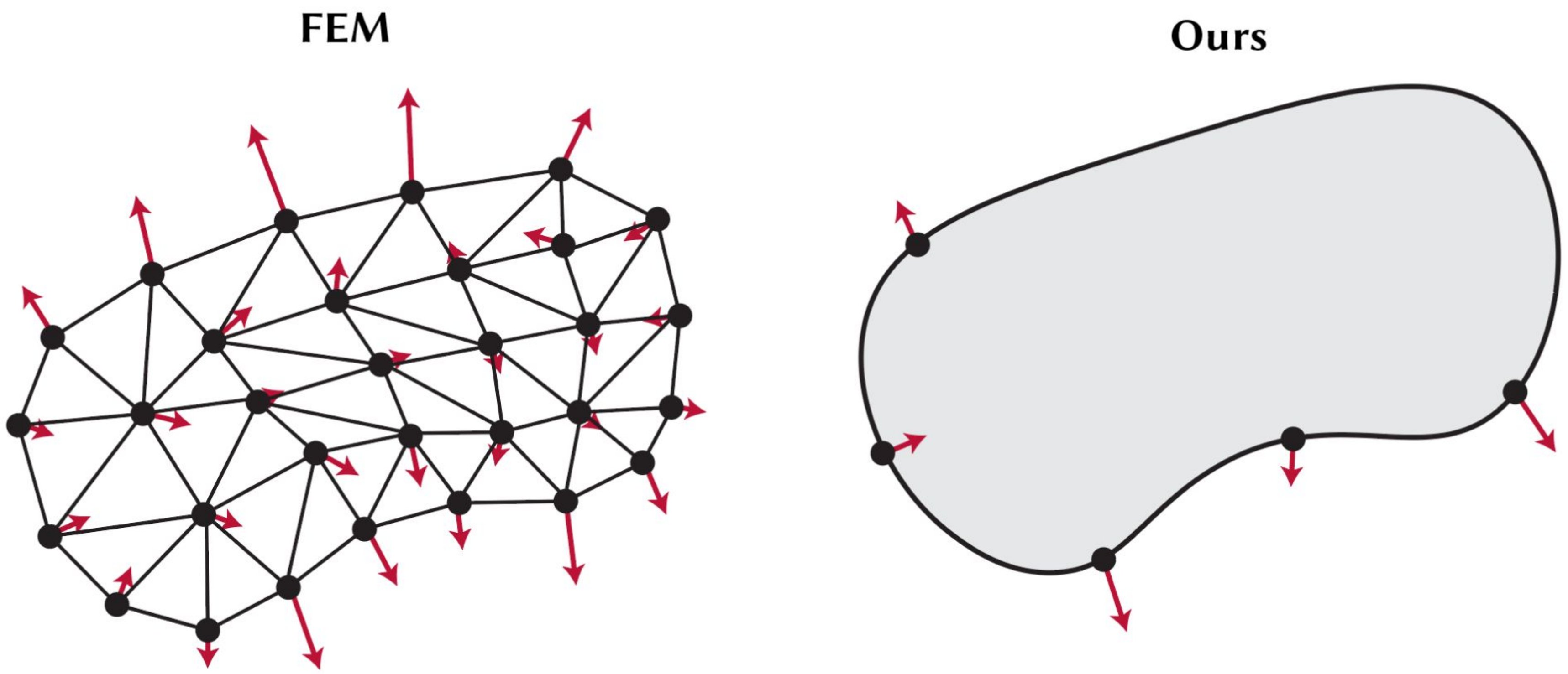
Elastic Wave Equation (Frequency Domain)

$$(\lambda + 2\mu)\nabla\nabla \cdot u - \mu\nabla \times (\nabla \times u) + \rho\omega^2 u + f = 0$$

Solvable!

Solving Elastic Wave

Elastic Wave Equation (Frequency Domain)



Overall Pipeline

$$\mathrm{f}(x, t)\ \Omega(x)$$

Fourier Transform

Elastic Solve

Set Neumann BC

Solve Radiation

Laplace Solve

Duffin Correspond.

Kelvin Trans.

Sound Render

$$\mathbf{x}$$

Kelvin Neumann BC

With the Neumann boundary condition

$$|\mathbf{y}|^2 \partial_{\mathrm{N}} U(\mathbf{y}) = \omega^2 \rho_{\mathrm{air}} u_n(\phi(\mathbf{y})), \quad \mathbf{y} \in \partial\Omega_{\mathrm{inv}} \tag{7}$$

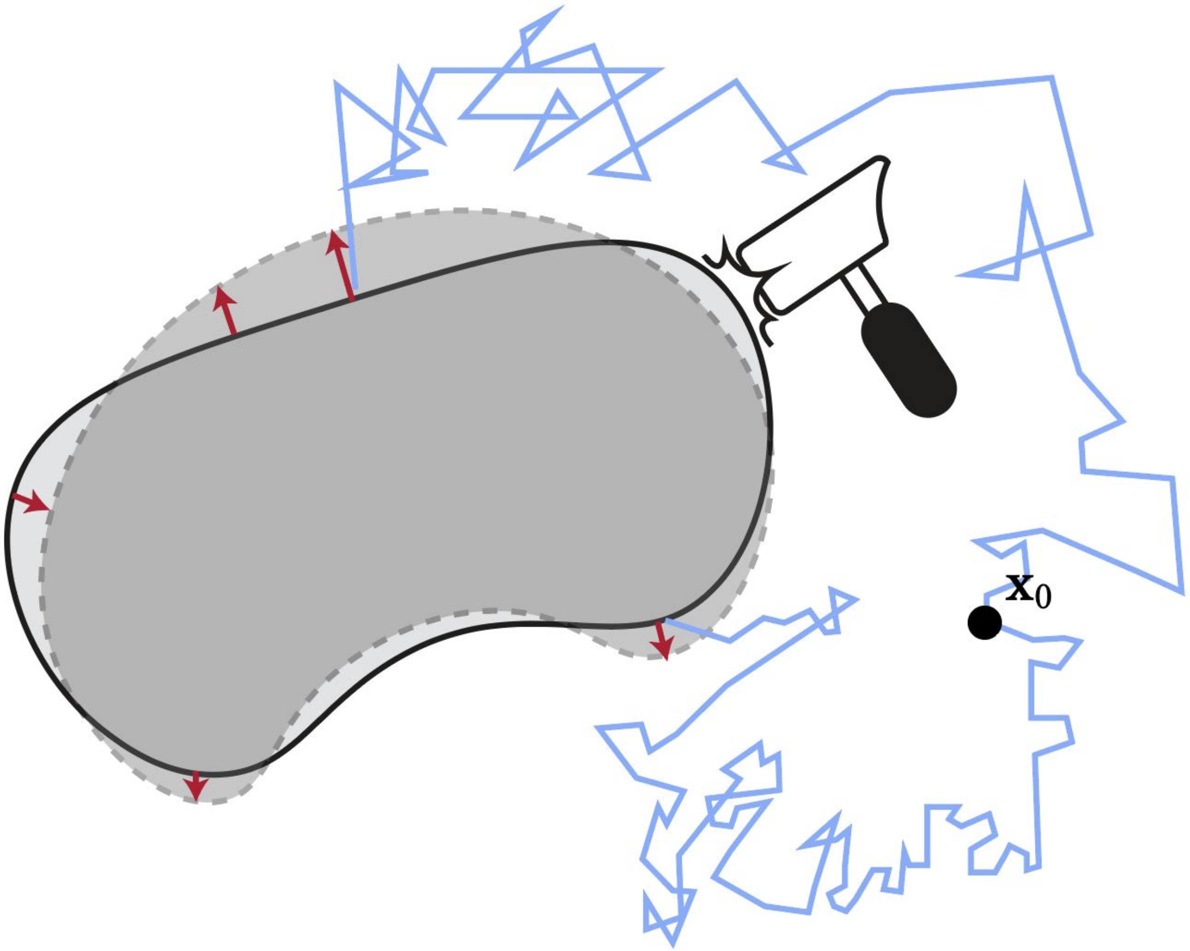
where $G(\mathbf{y}) = |\mathbf{y}| \exp i \frac{\omega}{|\mathbf{y}|}$. Solving the original equation is then equivalent to solving for $V(\mathbf{y})$:

$$\Delta V(\mathbf{y}) - 2i\omega \frac{\mathbf{y}}{|\mathbf{y}|^3} \cdot \Delta V(\mathbf{y}) = 0, \quad \mathbf{y} \in \Omega_{\mathrm{inv}} \tag{8}$$

with the Robin boundary condition

$$|\mathbf{y}|^3 e^{\frac{i\omega}{|\mathbf{y}|}} \partial_{\mathrm{N}} V(\mathbf{y}) + \mathbf{N} \cdot y \left(|\mathbf{y}| - i\omega e^{\frac{i\omega}{|\mathbf{y}|}} \right) V(y) = \omega^2 \rho_{\mathrm{air}} u_n(\phi(\mathbf{y})), \quad \mathbf{y} \in \partial\Omega_{\mathrm{inv}} \tag{9}$$

Overall Pipeline



Acceleration Techniques - Recycling Walks

- Once a frequency is solved, other frequencies can be cheaply evaluated with the cached walks

- Once cached, parallelly solve for all frequencies

- Bidirectional solving of Poisson equations

- Boundary Value Caching

Experiment

Experiment

Wavesolver Dataset [Wang, et al. 2018]

Scene	Wang el al. 2018 run-time (s)	Ours precompute (s)	Ours run-time (s)	Ours total (s)	**Speed-up**
Spolling Bowl	**3000**	2.12	0.647	**2.77**	**1083**
Wineglass	**3000**	5.62	6.64	**12.26**	**245**
LEGO	**3240**	0.803	1.67	**2.47**	**1312**

Conclusion

Summary

Solving Acoustic wave and Elastic wave equation in frequency domain via Duffin Correspondence. Our contribution includes:

- Deriving Neumann BC for Duffin's Correspondence
- Deriving Kelvin Transformed BC for Helmholtz Equation
- Efficient Implementation of Helmholtz solver on GPU

Expected Benefits

- Fast Wave Solver can use Real-time Physically based sound rendering

- It can use not only sound rendering, but also any situations wave equation used.
    - Seismic wave, schrodinger equation, maxwell equation, …

Future Works

- Implement Elastic Wave Solver
- Extend beyond rigid bodies (fires, water, etc.)
- Implement More Efficient Sampling (Bidirectional, Adaptive, etc.)

References

W.C. Chew: *Elastic Wave Lecture Notes (1991)*
R.J. Duffin: *Yukawan potential theory. J. Math. Anal. Appl. 35 (1971), 105–130.*